NAG Toolbox for MATLAB

f07fs

1 Purpose

f07fs solves a complex Hermitian positive-definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07fr.

2 Syntax

```
[b, info] = f07fs(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07fs is used to solve a complex Hermitian positive-definite system of linear equations $AX = B$, this function must be preceded by a call to f07fr which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = 'U', $A = U^{\mathrm{H}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{H}}Y = B$ and then $UX = Y$.

If **uplo** = 'L', $A = LL^{\mathrm{H}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{H}}X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'

$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of A, as returned by f07fr.

3: **b**(**ldb**,$*$) **– complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** $=$ 'U', $|E| \leq c(n)\epsilon \left|U^{\mathrm{H}}\right||U|$;
if **uplo** $=$ 'L', $|E| \leq c(n)\epsilon |L|\left|L^{\mathrm{H}}\right|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07fv, and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling f07fu.

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to f07fv to refine the solution and return an error estimate.

The real analogue of this function is f07fe.

9 Example

```
uplo = 'L';
a = [complex(3.23, +0), complex(0, 0), complex(0, 0), complex(0, 0);
       complex(1.51, +1.92), complex(3.58, 0), complex(0, 0), complex(0,
0);
        complex(1.9, -0.84), complex(-0.23, -1.11), complex(4.09, +0),
complex(0, 0);
       complex(0.42, -2.5), complex(-1.18, -1.37), complex(2.33, +0.14),
complex(4.29, +0)];
b = [complex(3.93, -6.14), complex(1.48, +6.58);
     complex(6.17, +9.42), complex(4.65, -4.75);
     complex(-7.17, -21.83), complex(-4.91, +2.29);
     complex(1.99, -14.38), complex(7.64, -10.79)];
[a, info] = f07fr(uplo, a);
[bOut, info] = f07fs(uplo, a, b)
```

```
bOut =
   1.0000 - 1.0000i  -1.0000 + 2.0000i
  -0.0000 + 3.0000i   3.0000 - 4.0000i
  -4.0000 - 5.0000i  -2.0000 + 3.0000i
   2.0000 + 1.0000i   4.0000 - 5.0000i
info =
           0
```